Relationships between Language Proficiency and Mathematics Achievement

Report

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Abstract

The present study is the first in a series designed to investigate relationships between English language proficiency and mathematics achievement for English learners (ELs) and the stability of those relationships across different student populations. In the current study we investigated the hypothesis that there are meaningful relationships across academic English language proficiency, as measured by ACCESS of ELLs®, and achievement on a US state’s mathematics tests. Two models were developed, one for grades 3-5 students, the other for grades 6-8 students. Both models had the same general pattern of relationships between English language proficiency and mathematics achievement. These results suggest that success in mathematics is influenced by English language proficiency in both productive (writing and speaking) and receptive (listening and reading) skills, with receptive skills being more closely associated with success on mathematics content tests. Receptive skills in both general and technical areas directly influence mathematics achievement. Both models show that it is not just proficiency in the language of mathematics that influences mathematics achievement - all but one of the ACCESS measures of the English language proficiency standards influence mathematics achievement. A construct, which we are calling a language construct, influences the general and technical productive skill and ultimately mathematics achievement. Our conclusion is that helping ELs with their technical and general language proficiency helps their mathematics achievement.

Context

In 2001 the US federal No Child Left Behind Act of 2001 and corresponding state statutes mandated that states annually administer a standards-based English language proficiency test to all English learners¹ (ELs) in kindergarten through grade twelve in public schools. Of particular interest to the study reported in this paper is the law’s focus on the relationship between students’ English language proficiency and their achievement on a state’s content tests.

In 2003 a consortium, known as the World-Class Instruction, Design and Assessment (WIDA), was established which by the end of that year consisted of eight US states. Currently there are 27 WIDA Consortium state members. The consortium aims to support academic language² development and academic achievement for linguistically diverse students through high quality standards, assessments, research, professional development, and enable consortium members to meet federal government accountability requirements.

By 2004 the WIDA English language proficiency standards³ and an accompanying assessment system had been developed to monitor students' progress in acquiring academic English. This work was based on the theory of academic language proficiency being the basis for academic achievement (Boals, Lundberg and Spalter, 2010). The WIDA standards reflect the classroom language of mathematics, science, social studies, language arts, and social and instructional language⁴. The assessment system, Assessing Comprehension and Communication in English State-to-State for English Language Learners (ACCESS for ELLs®⁵), is a secure large-scale English language proficiency assessment given to kindergarten through 12th graders who have been identified as ELs. ACCESS is aligned to the WIDA English language proficiency standards, is vertically scaled and assesses language proficiency in four language

¹ English learners are linguistically and culturally diverse students who have been identified through reliable and valid assessment as having levels of English language proficiency that preclude them from accessing, processing, and acquiring unmodified grade level content in English and thereby, qualifying for language support services. These students’ levels of English language proficiency limits their ability to meaningfully participate on state content assessments in English.

² Academic language is defined as the vocabulary, grammatical structures and discourse required in learning the academic content of school subjects; aspects of language strongly associated with literacy development and achievement.

³ English language proficiency standards are defined as criteria that express the language expectations of English learners at the end of their English language acquisition journey across language domains.

⁴ Social and instructional language is the everyday and instructional registers used in interactions outside and inside school.

⁵ Henceforth, ACCESS for ELLs® will be referred to as ACCESS.
domains: listening, speaking, reading, and writing. ACCESS was administered in 23 states to 824,919 students in 2010-2011.

Research basis

In his recent review of English learner policy and practice, Kenji Hakuta (2011) notes that content teachers should be engaged in the development of academic language and he expects "the coordination of English language proficiency standards and content will continue to be an issue ... since there is a strong agreement that all teachers need to think of themselves as teachers of academic English" (p. 171-172).

According to Schleppegrell (2007), language and learning a content area cannot be separated. Research has shown that poor academic language proficiency can impede ELs’ mathematics learning. For example, Parker et al. (2009) showed that English language proficiency reading and writing scores are significant predictors of reading, writing and mathematics scores on large-scale content assessments for 5th and 8th grade ELs.

We selected mathematics achievement for our research because relationships between academic language proficiency and mathematics are of particular interest to practitioners. Gee (2004) notes that while learning to read has been a major focus of educators and policy makers, this is not what is hard for students. He argues that “learning to read and learn in academic content areas like mathematics, social studies and science” (p. 3) with their specialist languages is the critical issue and that schooling is ultimately about learning these specialist languages linked to content areas.

With academic language proficiency being the theoretical foundation for WIDA’s English language proficiency standards and the accompanying assessment system, we surmised that ELs’ performance on written mathematics tests would be affected by their English language proficiency, as measured by ACCESS. When designing the study we wanted to reflect the complex realities of classrooms by comprehensively examining the interconnectedness across different language domains, English language proficiency in different content areas, and achievement on different state mathematics tests. This paper reports results from the first stage of a project aimed at developing a set of models designed to examine the stability of relationships between English language proficiency and mathematics achievement across different EL populations. Our hypothesis was that:

There are structural linear relationships between ELs’ proficiency in the academic languages of mathematics, science, language arts and social studies, and social and instructional language (as measured by ACCESS) and their achievement on one state’s mathematics tests.

Method

Data sources (ACCESS and the state mathematics tests)

A dataset became available in which ELs in two grade clusters, 3-5 and 6-8, participated in the same ACCESS assessment form (i.e. tier B items\(^6\)) in February/March 2008 and the state’s mathematics tests in October/November 2008. There were 613 ELs in grades 3-5 and 560 in grades 6-8. The data were the ACCESS measures of WIDA’s English language proficiency standards and the state’s four mathematics measures: number and operations; functions and algebra; geometry and measurement; and data, statistics and probability.

\(^6\) The original dataset had ACCESS results from four tiers: K, A, B, and C. While the tiers overlap, they keep the test shorter and more appropriately targeted to each student’s range of language skills.
The arrangement of the English language proficiency standards within the language domains is shown in Table 1. Note that the English language proficiency standards combine listening and reading in each of the five content areas, but writing and speaking standards are separate. In the speaking domain some standards combine different content areas i.e. mathematics and science, and language arts and social science. Also, ACCESS assesses the language proficiency in the technical areas of mathematics and science\(^7\), as well as the non-technical areas of language arts, social studies, and social and instructional language.

**Table 1: ACCESS variables: English Language Proficiency Standards across Language Domains**

<table>
<thead>
<tr>
<th>Language domain elements</th>
<th>English language proficiency standards</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematics</td>
</tr>
<tr>
<td></td>
<td>Technical skills</td>
</tr>
<tr>
<td>Listening/reading</td>
<td>✓</td>
</tr>
<tr>
<td>Writing</td>
<td></td>
</tr>
<tr>
<td>Linguistic complexity(^a)</td>
<td>✓</td>
</tr>
<tr>
<td>Language control(^b)</td>
<td>✓</td>
</tr>
<tr>
<td>Vocabulary usage(^c)</td>
<td>✓</td>
</tr>
<tr>
<td>Speaking</td>
<td>✓</td>
</tr>
</tbody>
</table>

*Note:* \(^a\) *Linguistic complexity* is the amount and quality of writing in a given situation. \(^b\) *Language control* is the comprehensibility of the communication level based on the amount and types of errors. \(^c\) *Vocabulary usage* is the specificity of words or phrases for a given context. (Gottlieb, Cranley & Oliver, 2007.)

The state’s mathematics tests are written tests which contained a combination of multiple choice and short answers items. Short answer items ask students to generate a short response to a question. The grades 5 to 8 tests contained constructed response items for more complex items and required students to give a longer response.

**Analytical approach**

Because of the interdependence of the measurements within ACCESS and within the mathematics tests, structural equation modeling was selected as the most appropriate statistical tool with which to investigate relationships between English language proficiency and academic achievement. The aim is to identify models which best fit the data and which practitioners and researchers consider make the most sense and reflect the realities of the classroom.

One of the advantages of structural equation modeling is that it enables you to identify an underlying construct(s) (or factor or latent variable) from several observed (or measured) variables and then examine relationships between those constructs. This set of constructs, with the observed variables, and relationships (as indicated by paths) becomes the model. The assumption is that a set of observed variables is a linear combination of the underlying construct(s).

\(^7\) For this study of language proficiency and mathematics achievement, technical language skills are described by the languages of mathematics and science.
When developing these models, some variables are omitted because their measurement errors are too large, thereby detracting from the model fit. Some relationships between constructs are omitted because they are not statistically significant or do not make sense given previous experience, theory or research. Various statistics are used to identify the best fitting models. For example, standardized path coefficient statistics indicate the relative strengths of the relationships between constructs.

Results and discussion

Preliminary evidence supporting our hypothesis was found in the state’s mathematics data. Relationships in each grade cluster model show the same general pattern and all path coefficients between the constructs are statistically significant. A schematic representation of these relationships is shown in Figure 1. (Attachments A and B provide statistical details for each model. Abbreviations and symbols used for the variables and constructs are listed in Attachment C.)

Figure 1: Language proficiency and mathematics achievement relationships (no. 1).

The models suggest that:

- success in mathematics is influenced by English language proficiency in both productive (writing and speaking) and receptive (listening and reading) skills, with receptive skills being more closely associated with mathematics success on the state’s mathematics tests
- receptive skills in both general and technical areas directly influence mathematics achievement
- general and technical productive skills in writing and speaking play a part in mathematics achievement
- writing skills in the technical areas of mathematics and science are more closely related to mathematics achievement than the non-technical areas of social studies and language arts and speaking
- all but one\(^8\) of the ACCESS measures of the English language proficiency standards influence mathematics achievement, not just proficiency in the language of mathematics or the particular language domain of writing.

\(^8\) To improve the fit of the models, the ACCESS measure of listening and reading in social and instructional language was dropped.
Both the grades 3-5 and 6-8 models show strong relationships between the reading and listening skills construct and mathematics achievement. The variance in mathematics achievement explained by these general and technical receptive skills is between 31% and 34%.

Fundamental to all of the relationships in the models is an underlying construct we are calling a language construct. This construct strongly influences general writing skills and ultimately mathematics achievement. Path coefficients of around 0.55 between the language construct and general writing skills are amongst the highest coefficients of all the paths in the models. The only higher path coefficient is between the reading and listening constructs and mathematics achievement.

Consistent with the theory of academic language proposed by Boals, Lundberg and Spalter (2010), the underlying language construct may be capturing components of academic language proficiency influenced by student characteristics and competencies, the degree of cognitive engagement, instructional supports, the linguistic complexity of tasks, and instructional contexts. Alternatively, the construct may be capturing more general characteristics, such as strategic competence9 or other cognitive or meta-cognitive strategies which underpin performance on mathematics tests more broadly. On the other hand, the language construct may be an artifact of the way in which the ACCESS measures and mathematics tests are constructed.

Another way of conceptualizing our findings is shown in Figure 2. This diagram shows the language construct being the foundation which influences general and technical language skills. These skills which comprise both receptive and productive skills subsequently influence mathematics achievement.

Figure 2: Language proficiency and mathematics achievement relationships (no. 2)

Limitations

While the models demonstrate links across different aspects of English language proficiency and mathematics achievement, care should be taken not to generalize the models to other populations and other curriculum areas. The results are based on 53% to 76% of students who participated in the same ACCESS assessment form (i.e. tier B items), but only 23% to 25% of students who participated in all ACCESS assessment forms (i.e. tiers A, B, C and K). Furthermore, the results are from one content area, mathematics, in one state, and students from one year. (Refer to Attachment D for a diagram showing how the sample sizes were obtained from the original dataset.) Sample sizes precluded analyses being conducted for each grade level or students with different language backgrounds. Furthermore,

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9 Strategic competence is defined as a set of meta-cognitive strategies that manage the ways in which students use different attributes to interact with the characteristics of the language use situation.
the assumption is that relationships amongst the constructs of English language proficiency and mathematics achievement are manifested in the same way across the grade levels within a model and ELs with different linguistic backgrounds. Also, the reader should note the seven or eight month delay between the students' English language proficiency being assessed and the same students sitting the mathematics tests.

Conclusions

The models highlight the complexities of the interconnectedness of proficiency in academic languages, social and instructional language, the language domains, and performance on written mathematics tests. Success in mathematics is supported not only by proficiency in the academic language of mathematics, but also by proficiency in academic languages of other content areas, social and instructional language and in all four language domains. At this stage of the research project we therefore suggest that:

> Helping ELs with their English language proficiency in technical and general areas across all areas of the curriculum, including mathematics, helps their mathematics achievement. Taking a holistic approach to language acquisition is important for their achievement in mathematics.

The findings provide preliminary empirical evidence of an underlying construct that directly relates to proficiency in general and technical writing and speaking skills. However, further research is needed to ‘tease out’ the stability and nature of this underlying construct in different contexts and whether it is related to academic language proficiency or strategic competence or another construct.

Acknowledgements

The authors wish to thank the staff of the state department of education for providing the data set and for their professional advice; Drs. Dorry Kenyon and David MacGregor (Centre for Applied Linguistics), Prof. David Kaplan (University of Wisconsin-Madison) and Dr. Margaret R. Meyer (University of Wisconsin-Madison), and Drs. Margaret Wu and Ute Knoch (University of Melbourne) for generously giving their time and suggestions during this research.

References


Model: grades 3-5, tier B.

Indirect Effects

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R-SQUARE

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Path coefficients: *p<0.05; **p<0.01

N=613

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<td>F5 BY</td>
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F1 on  F2  F4  F5;
F4 on  F3  F6;
F5 on  F3  F6;
F7 on  F1 ;
ALP by  F2  F3  F6;

MODELFIT INFORMATION

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N=613

Language Construct

Writing LA/SS F6

Math Achiev F7

Writing SI F3

Listening/Reading F1

Writing Math F4

Writing Sci F5

Speak F2
Model: grades 6-8, tier B.

R-SQUARE

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DF=0.71**

Effects from ALP to F7

| Total indirect | 0.203 | 0.028 | 7.273 | 0 |

Indirect effects

| Indirect effects | STDYX Standardization | |
|------------------|------------------------|-
|                  | Estimate | S.E. | Est./S.E. | P-Value |
|                  |          |     |            |         |

Effects from ALP to F7

| ALP by F2 F3 F6; | 0.32** |
| F7 on F1 | F5 on F3 F6; |
| F7 on F1 ; ALP | F2 F3 F6; |

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RMSEA

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CFI/TLI

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SRMR

| Value | 0.05 |

N=560

Path coefficients: *p<0.05; **p<0.01
Abbreviations and Symbols used for the Variables and Constructs

**ACCESS Variables and Abbreviations used in the Models in Attachments A and B**

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<tr>
<th>Language domain elements</th>
<th>English language proficiency standards</th>
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<td></td>
<td>Mathematics</td>
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<td>Technical skills</td>
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<td>Listening/reading</td>
<td>Comp_Ma</td>
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<td>Writing</td>
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<tr>
<td>Linguistic complexity$^a$</td>
<td>Wr_Ma_Cx</td>
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<tr>
<td>Language control$^b$</td>
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<td>Vocabulary usage$^c$</td>
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<td>Speaking</td>
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</table>

*Note: $^a$ Linguistic complexity is the amount and quality of writing in a given situation. $^b$ Language control is the comprehensibility of the communication level based on the amount and types of errors. $^c$ Vocabulary usage is the specificity of words or phrases for a given context. (Gottlieb, Cranley & Oliver, 2007.)*

**State Mathematics Variables and Abbreviations used in the Models in Attachments A and B**

Mathematics scores for these variables were converted to t-scores (mean of 50 and standard deviation of 10).

<table>
<thead>
<tr>
<th>Standardized variables</th>
<th>Abbreviations</th>
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<td>Functions and algebra</td>
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**Labels for constructs used in Figure 1 and Attachments A and B**

<table>
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<th>Language skills</th>
<th>Constructs</th>
<th>Abbreviations</th>
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<td>Productive skills</td>
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<td>Speak</td>
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<td>Writing LA/SS</td>
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<td>State mathematics</td>
<td>Higher order construct</td>
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</table>
Diagram showing how the sample sizes for the modeling were obtained from the original dataset with 9,618 records

The objective was to obtain 2 datasets (grades 3-5 and grades 6-8) which contained complete records for all ACCESS tier B and mathematics results.